AN INTEGRATION OF FDI AND DX APPROACHES TO POLYNOMIAL MODELS

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Abstract: In engineering applications, many models are a set of polynomial constraints. In order to automate and improve the diagnosis of these models, we propose a new approach for the integration of both FDI and DX approaches. It allows us to achieve a synergy that produces results that could not be obtained if each one was operating individually.

This paper uses Gröbner bases to generate a more single model of the systems. First, it eliminates the non-observable variables of the constraints of the model and afterwards, we construct a context network with the minimal possible conflicts and polynomial constraints in the nodes of this network. Our methodology proposes an algorithm that uses different aspects of FDI and DX approaches to obtain the minimal diagnosis. This novel approach may be very useful for on-boarding diagnosis.

Keywords: Diagnosis, Polynomial models, Model-based diagnosis, Gröbner bases.

1. INTRODUCTION

The faults produced in components and processes can cause undesirable stops and damage in the systems, with the consequent cost increase and production decrease. It is also necessary to take into account that these faults can produce a considerably negative impact in the environment, which has to be avoided. Therefore, in order to keep the systems within the desired security, production and reliability levels, some methods which allow the detection and diagnosis of the faults produced in the systems have to be developed. The fulfillment of these economic and environmental demands will allow the companies to remain in a more and more competitive market.

Diagnosis allows to identify the parts which fail in a system. It generally integrates the monitorization (sensors are supposed to work correctly) and the diagnosis (fault detection and identification). A diagnosis task determines why a correctly designed system does not work as it was expected. The explanation of such a wrong behavior from a determined observation is the main task of diagnosis. Most approaches produced in the last decade to carry out diagnosis have been based on the use of models (FDI and DX approaches). These models are based on the knowledge of the system to diagnose, which can be formally well structured and in agreement with well-known theories, or can be known by means of an expert's experience and data of the system or process. Sometimes, a combination of both types of information can also be presented.

In the area of Artificial Intelligence, the first work related to diagnosis was presented with the objective of identifying faults in the component systems, based on the structure and its behavior [4]. The first implementations to perform diagnosis were DART [10] and GDE [6], which used different inference mechanisms to detect possible faults. Diagnosis formalization was presented in [17] [7], where a general theory was proposed for the problem of explaining the discrepancies between the observed and correct behaviors that the mechanisms subject to the diagnosis process (logical-based diagnosis) have. Based on this, most DX approaches for components characterise the diagnosis of a system as a collection of minimal sets of failing components which would explain the observed behaviors (symptoms). The importance of having a good model in DX can be deduced from this. This kind of diagnosis can correctly diagnose the important parts of the component systems that are failing.

The diagnosis in DX examines the models to determine the cause of the fault. The models centered on mechanisms describe the systems by means of input-output relations. Many diagnosis methods use models centered on mechanisms which require engineers to develop models and thus avoid the building of fault models, besides the normal operation model. Building fault models in a system is useful when the faults are well-known and easy to model. However, this limits the diagnosis of the system to well-known faults. An exhaustive revision of the approaches about diagnosis task automation can be found in [8], and for a discussion of model-based diagnosis applications [2] can be consulted.

Another research community dealing with diagnosis of systems is the fault detection and Isolation one (FDI), and there are different surveys about it [15] [12]. Redundant relations within the behavior model (redundancy-based diagnosis) are used in this approach. In large scale process models, sometimes these relations are not found or relations of different kinds coexist. Then, an approach consisting in a structural analysis which uses the graph theory is proposed [16]. The integration of FDI theories with the DX community ones and their equivalence proofs have been shown for several hypothesis in a recent work [3].

Our work presents a new approach which integrates both approaches with the main idea of improving and automating the diagnosis process in systems whose models consist of polynomial equality constraints. For this, symbolic processing algorithms (Gröbner bases) of the initial model are used, and they generate the minimal possible conflict sets of the model according to its structure and behavior. This new model will be treated by the corresponding algorithms with the object of promoting the advantages presented by each one of the FDI and DX approaches. The DX approach allows us to calculate the minimal possible conflict context (MPCC) and multiple minimal diagnosis and FDI approach finds out the single faults using previous MPCCs.

A proposition in DX [16] related to our work about the objectives, although not to the method, presents the concept of a possible conflict as an alternative to the use of pre-compiled dependency-recording. A method is developed to calculate the minimal chains which can be evaluated, and the minimal models which can also be evaluated. The previous use of Gröbner bases in the FDI community is proposed in previous works [9] and [11]. Our work differs from other works because it proposes the integration of both kinds of approaches, in order to achieve a synergy that produces results that could not be obtained if each one were operating individually. Therefore, our approach uses concepts from both communities (DX and FDI), such as context network, analytical redundancy constraint, possible conflicts and hitting set. Several important advantages are achieved with this integration: (1) reduction of the model of the system to diagnose, (2) hierarchy and more high level of abstraction of the model and (3) knowledge of the separable components. The result is an automatic and very efficient method to diagnose systems with polynomial models.

Our paper has been organised as follows: first, an example to carry out the diagnosis process is presented; next, the necessary definitions and notations which allow to formalize the subsequent operations that will be carried out are exposed. Sections 4 and 5 show the way of obtaining the new model from the original by means of Gröbner bases, the data structures and algorithms which lead to an efficient minimal diagnosis for a defined observation. Finally, the results for a more complex example, our conclusions and the future works in this research line are presented.

2. A WELL-KNOWN EXAMPLE

A very often used example in the bibliography concerning model-based diagnosis [4], [10], [7], [2] and [16] is the one formed by three multipliers and two adders, as it is presented in figure 1.

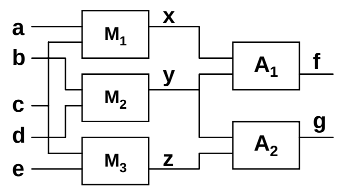

Fig. 1. Circuit formed by three multipliers and two adders

The multipliers are represented in figure 1 as M_1, M_2 and M_3, and the adders as A_1 and A_2. In this system, the component or components that are failing have to be identified, taking into account that the only observable values are the ones represented as a, b, c, d, e, f and g.

3. DEFINITIONS AND NOTATION

The definitions and notation used are based on the concepts developed in the diagnosis community, based itself on the logic (DX) and on redundancies (FDI). The objective is that the synergy of both approaches will produce diagnosis results which are as representative as possible of what is happening in the system in the shortest time.

As it has already been mentioned, model-based diagnosis requires a system model. In our case, we will only deal with the case which has a model of system constraints that derives from its own structure, and which has links between components (structural model) and the behavior of each model component. With this model and with the idea of formalizing the diagnosis process, the definitions and notation used in the development of this work need to be exposed.

Definition 1. The System Polynomial Model (*SPM*)*:* It can be defined as a finite set of polynomial equality constraints *P* which determine the system behavior. This is done by means of the relations between the system non-observable variables (V_{nob}) and the input-output observable variables (V_{io}) which are directly obtained from sensors that are supposed to work correctly. Then, the following tuple for a system polynomial model is obtained *SPM* (P, V_{io}, V_{nob}).

For the example presented in Section 2, the SPM represented in Table I will be obtained.

Table I. System Polynomial Model of the system of three multipliers and two adders

Polynomial Constraints	V_{io}	V_{nob}
M_1: x=a*c	a = a_{ob}	x
M_2: y=b*d	b = b_{ob}	y
M_3: z=c*e	c = c_{ob}	z
A_1: f=x+y	d = d_{ob}	
A_2: g=y+z	e = e_{ob}	
	f = f_{ob}	
	g = g_{ob}	

Definition 2. Observational Model (*OM*): A tuple which assigns values to the observable variables. In the example proposed in Section 2, an observation set may be:

$$MO \equiv \{a_{ob}=2, b_{ob},=2, c_{ob}=3, d_{ob}=3, e_{ob}=2, f_{ob}=10, g_{ob}=12\}$$

Definition 3. Context Set (*CS*): A context set of a SPM is a collection of component sets which compose the system. In the case of the circuit represented in figure 1, it will be:

$$CS \equiv \{A_1, A_2, M_1, M_2, M_3, \{A_1,A_2\}, \{A_1, M_1\}, ...\}$$

and, thus, the possible context set will be 2^{comp}, where *comp* is the number of components which represent the behaviour model of the system.

Definition 4. Diagnosis Problem (*DP*): It can be defined by means of a tuple formed by a System Polynomial Model and an Observational Model. The result of this problem will be a set of elements that belong to the set of the system faults which reflect in a minimal way the information of the possible failing components *DP*(*SPM,OM*). To obtain these components, it is necessary to define the following concepts:

Definition 5. Context Network: A graph formed by all the elements of the context set of the system according to the way proposed by ATMS [5]. Figure 2 represents the context network for the problem presented in Section 2. In our work this context network will be enriched with the *Context Analytical Redundancy Constraints*.

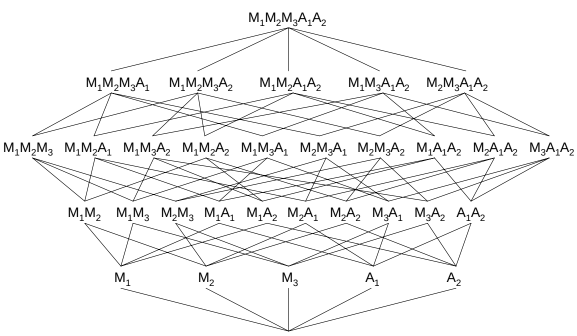

Fig. 2. Context network for the problem of three multipliers and two adders

Definition 6. Context Analytical Redundancy Constraint (*CARC*): Set of constraints derived from SPM and in such a way that only the observed variables are related. In this work, we are only dealing with the models in which it is defined by polynomial equality constraints. In these constraints, their truth value can be evaluated from the system observed variables through the corresponding monitorization.

Definition 7. Minimal Possible Conflict Context (*MPCC*): The conflict contexts are those contexts of the context network which have an inconsistent result when the verification of the CARCs is carried out. A context is minimal possible context when its sub-contexts are not conflict context.

The use of these minimal conflict contexts allows to establish a proposition for the minimal diagnosis as in [17]. Considering a system model, an observation and the system component set, a diagnosis is minimal if and only if it is a set affected in a minimal way (hitting set) for the collection of minimal conflict contexts. A "hitting set" for a collection of sets is the one which intersects any set of the collection of minimal conflict sets. Some of the previous operations can be carried out off-line, although they are generally carried out on-line.

4. THE CONTEXT ANALYTICAL REDUNDANCY CONSTRAINTS OF THE MODEL

The model which reflects the system structure and behavior presents the constraints that link the system inputs and outputs, but many times some intermediate variables appear which are not observable and which do not allow to determine whether there are faults in the components in a direct way. The intuitive idea of getting the system constraints is trying to reflect in it the structural aspects, in such a way that the component sets that are not interrelated will not be verified.

Therefore, in our work, this system constraint set will be submitted to a set of symbolic algorithms which will transform these constraints in order to get what has been defined as context constraints. For this reason, Gröbner bases will be used, since the model constraints are polynomial equality constraints.

4.1. Gröbner Bases

Gröbner bases theory is the origin of many symbolic algorithms used to manipulate multiple variable polynomies. For an introduction to Gröbner bases [1] and [13] can be consulted. The algorithm used for the system polynomial equations is based on the ideas proposed by Buchberger [2]. This algorithm is a generalization of Gauss' elimination for multivariable lineal equations and of Euclides' algorithm for one-variable polynomial equations. One of Gröbner bases has better computational properties than the original system. It can be said that it is very easy to determine if the system can be solved.

The main idea is to transform the polynomial constraint set into a standard form for the resolution of problems. Having the set of equality polynomial constraints of the form P=0, Gröbner bases produce an equivalent system G=0, which has the same solution as the original, but it is generally easier to solve.

Concerning the advantages that the use of Gröbner bases has for the system models subject to diagnosis, it can be said that:

- If the model is over-restricted and has redundant equations, these redundancies will disappear when a reduced Gröbner base is calculated.
- If the model is over-restricted and inconsistent, one of the constraints which provides the algorithm will be 1=0, that is obviously inconsistent.
- If the model is infra-restricted, the new model also provides useful information for its subsequent resolution.

For our work, there is a function, which has been called *GröbnerBase*. It calculates Gröbner bases in such a way that if there is a system to diagnose, represented by an *SPM* formed by a finite set of polynomial equations, a set of observable variables and a set of non-observable variables, the constraint set of the different contexts is obtained.

Let us consider, for instance, the context represented by the components $M_1M_2M_3A_1A_2$. Then, the *GröbnerBase* function receives the three previous sets. Thus, for the function *GröbnerBase* ($\{M_1M_2M_3A_1A_2\}$, $\{a,b,c,d,e,f,g\}$, $\{x,y,z\}$), the result would be the following system of polynomial constraints $\{b*d+c*e-g=0, a*c-c*e-f+g=0\}$.

The application of this function to the different contexts of a particular model will allow the building of the context network directed by constraints.

4.2. Building our Reduced Context Network

The reduced context network is obtained by means of a search algorithm directed by symbolic constraints. For the problem in Section 2, a new constraint network like the one represented in figure 3 is obtained by applying the corresponding *GröbnerBase* function to each context of the context network presented in figure 2.

This outline of the context constraints shows the constraints that must be satisfied for each node of the context network. Moreover, it shows that most three component nodes and all the two component nodes of the context network will not be catalogued as MPCC. Therefore, the search algorithm will avoid the computational treatment of them. This will improve the efficiency in the search of possible conflicts.

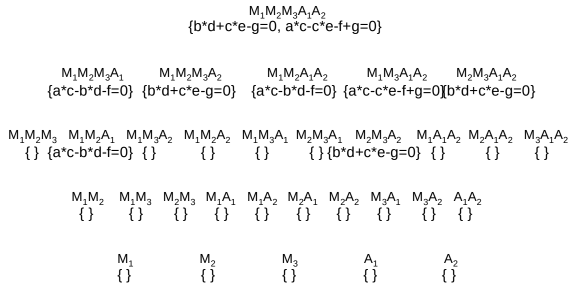

Fig. 3. Context network with symbolic constraints

5. OUR APPROACH FOR THE DIAGNOSIS

The method to carry out the diagnosis applies a search algorithm directed by the symbolic constraints previously obtained, with the idea of obtaining the possible minimal possible conflict contexts. The minimal diagnosis is obtained by means of the application of compiled rules using the fault signature from FDI for single faults and the hitting sets from DX for multiple faults.

5.1. Determination of minimal possible conflict contexts

The search of conflict sets is carried out through a search algorithm directed by the symbolic constraints of the minimal possible conflict contexts. In order to apply this search we use the *MPCCs* obtained in section 4.2. These *MPCCs* have associated an index to carry out a more efficient search process: $\{b*d+c*e-g=0, 1\}$, $\{a*c-c*e-f+g=0, 2\}$, $\{a*c+b*d-f=0, 3\}$.

The context network for the search of minimal possible conflict contexts is represented in figure 4. The numbers at the bottom of each context are the constraint indexes which correspond to this context. In order to determine the minimal conflict contexts, only the graph has to be traversed from the leaf nodes to the root node, in such a way that, if any of the upper contexts has the same constraint numbers of the previous ones, these are not considered as a MPCC.

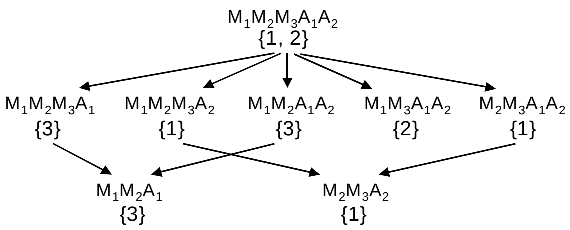

Fig. 4. Reduced context network.

Thus, it can be observed that when the graph is traversed from the context $M_1M_2A_1$, the preceding contexts $M_1M_2M_3A_1$ and $M_1M_2A_1A_2$ will not have to be considered, since they have the same constraint index Therefore, the context $M_1M_2A_1$ is a minimal possible conflict context. The same happens with the contexts $M_2M_3A_2$ and $M_1M_3A_1A_2$, in which the upper contexts have the same constraint index associated and, therefore, they will not be minimal possible conflict contexts. Therefore, the number of minimal possible conflict contexts in this model has been reduced to the contexts $M_1M_2A_1$, $M_1M_3A_1A_2$, $M_2M_3A_2$. These results have also been obtained in a recent work [16], but it uses a different methodology.

5.2. Determination of Minimal Diagnosis

To study two particular cases of diagnosis, there are two observational models:

$MO_1 \equiv \{a_{ob}=3, b_{ob}=2, c_{ob}=2, d_{ob}=3, e_{ob}=3, f_{ob}=10, g_{ob}=12\}$
$MO_2 \equiv \{a_{ob}=3, b_{ob}=2, c_{ob}=2, d_{ob}=3, e_{ob}=3, f_{ob}=10, g_{ob}=10\}$

If we want to find the single faults we will use the signature matrix (FDI) for the MPCCs of this system:

	F_{A1}	F_{A2}	F_{M1}	F_{M2}	F_{M3}
MPCC1	0	1	0	1	1
MPCC2	1	1	1	0	1
MPCC3	1	0	1	1	0

The minimal diagnosis consists of comparing the observation signature and the faults signature. If we want to find the multiple faults we will use the mentioned *MPCCs*. For the two observational models, we verify the satisfaction of the constraints described in the MPCCs. According to the DX approach, we obtain the minimal conflicts sets shown in Table III.

Table III. Minimal Conflicts Contexts and Multiple Minimal Diagnosis in two observational models

Observational Model	MPCCs	Multiple Minimal Diagnosis
MO_1	2,3	$\{A_2,M_2\},\{M_2,M_3\}$
MO_2	1,3	$\{A_1A_2\},\{A_1,M_3\},\{A_2,M_1\},\{M_1,M_3\}$

The diagnosis of multiple faults has an off-line part that uses the minimal context network in order to compile a set of rules from a combination of the single faults' rules. The on-line part matches the observational model with the left side of these rules, and the corresponding consequent will be the multiple minimal diagnosis. For the observational models previously shown, the obtained results are presented in Table III. Most part of the process is performed in an off-line, automatic and very efficient way. Thus, by using Gröbner bases, a reduced context network is produced.

6. A MORE COMPLEX EXAMPLE: A SYSTEM OF HEAT EXCHANGERS

The methodology has been applied to more complex systems, like the one shown in figure 6 from [13]. In this system, consisting of six heat exchangers, three flows f_i come in at different temperatures t_i.

The normal functioning of the system can be described by means of polynomial constraints, coming from three kinds of balances:

$$\sum_i f_i = 0 : \text{mass balance at each node,}$$

$$\sum_i f_i \cdot t_i = 0 : \text{thermal balance at each node,}$$

$$\sum_{in} f_i \cdot t_i - \sum_{out} f_j \cdot t_j = 0 : \text{enthalpic balance for each heat exchanger.}$$

The resulting system, thus, consists of 34 equations and 54 variables, from which 28 are observable: f_{11}, f_{12}, f_{13}, f_{16}, f_{17}, f_{18}, f_{19}, f_{112}, f_{21}, f_{26}, f_{27}, f_{212}, f_{31}, f_{33}, t_{11}, t_{12}, t_{13}, t_{16}, t_{17}, t_{18}, t_{19}, t_{112}, t_{21}, t_{26}, t_{27}, t_{212}, t_{31} and t_{33}. There is no direct measure of the rest of the variables. This defines three different subsystems, each one formed by two exchangers: {E1, E2}, {E3, E4} and {E5, E6}. Each of the six exchangers and each of the eight nodes of the system are considered as components to verify their correct functioning. Therefore, the resulting context network has a total of 2^{14} context sets.

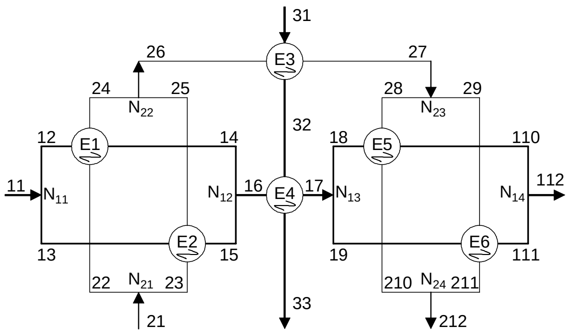

Fig. 6: System of heat exchangers

The Gröbner bases obtained for the complete system is composed by 14 polynomials (see Table V). These will be measured to detect the correct functioning of the system. If any of the CARCs is violated, the system fails, and it will determine which of the components could be the cause of the fault.

Table V. Minimal conflict constraints for the system

	CARCs
1	$f_{11} - f_{12} - f_{13} = 0$
2	$-(f_{11}\, t_{11}) + f_{12}\, t_{12} + f_{11}\, t_{13} - f_{12}\, t_{13} = 0$
3	$f_{17} - f_{18} - f_{19} = 0$
4	$-(f_{17}\, t_{17}) + f_{18}\, t_{18} + f_{17}\, t_{19} - f_{18}\, t_{19} = 0$
5	$f_{26} - f_{27} = 0$
6	$f_{16} - f_{17} = 0$
7	$f_{31} - f_{33} = 0$
8	$f_{16}\, t_{16} - f_{16}\, t_{17} + f_{26}\, t_{26} - f_{26}\, t_{27} + f_{31}\, t_{31} - f_{31}\, t_{33} = 0$
9	$-f_{12} - f_{13} + f_{16} = 0$
10	$-f_{18} - f_{19} + f_{112} = 0$
11	$f_{21} - f_{26} = 0$
12	$f_{27} - f_{212} = 0$
13	$f_{12}\, t_{12} + f_{13}\, t_{13} - f_{12}\, t_{16} - f_{13}\, t_{16} + f_{21}\, t_{21} - f_{21}\, t_{26} = 0$
14	$f_{18}\, t_{18} + f_{19}\, t_{19} - f_{18}\, t_{112} - f_{19}\, t_{112} + f_{27}\, t_{27} - f_{27}\, t_{212} = 0$

In figure 7, the context network is shown, where only the CARCs appear, together with their corresponding contexts. It can be observed how the previously mentioned subsystems are included in accordance with the observable variables. This separation is obtained automatically by our system.

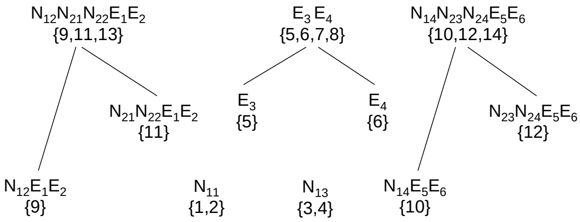

Fig. 7. Minimal Context Network for the system

Each CARC is related to one rule for the diagnosis, corresponding to a single fault in one of the components of the associated context that it appears first, starting from the leaves. So, if CARC 9 is violated, the corresponding rule indicates that there is a simple fault in N_{12}, E_1 or E_2. If several CARCs are violated, the minimal diagnosis is found by the hitting sets. But notice that this is done off-line, and only the combinations of the contexts in the same subtree, and without a precedence relation, must be considered. For this example, we obtain the following rules:

$$(9 \text{ and } 11) \rightarrow E_1 \text{ or } E_2 \text{ or } (N_{12} \text{ and } N_{21}) \text{ or } (N_{12} \text{ and } N_{22})$$
$$(10 \text{ and } 12) \rightarrow E_5 \text{ or } E_6 \text{ or } (N_{14} \text{ and } N_{23}) \text{ or } (N_{14} \text{ and } N_{24})$$

If the CARCs belong to different subtrees, the rules are obtained as conjunctions of the left and right side of the single fault rules. If a CARC is predecessor of another one, the first will not be considered.

7. CONCLUSIONS AND FUTURE WORKS

This work defines a new framework for the integration of FDI and DX approaches using Gröbner Bases. These symbolic techniques have been applied in order to obtain a more reduced and compiled knowledge of the system model subject to diagnosis.

We consider that it can be a reason for future research to take into account different truth qualitative values between 0 and 1 to point out the degree of the constraint satisfaction, and thus showing, in a certain way, the preferences of some minimal diagnosis. The extension of all this to the dynamic systems can be considered as the final objective of this research line.

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