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Railway interlocking systems and Gröbner bases

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Abstract

Railway interlocking systems are designed to prevent conflicting actions (related to the position of switches and signals) during everyday railway exploitation. A decision model (independent from the topology of the station) based on the use of polynomial ideals and Gröbner bases is presented. This decision model can also be used to check whether a given section is accessible by a train located in another section or not. The fact that trains could occupy more than one section does not affect the model. ©2000 IMACS/Elsevier Science B.V. All rights reserved.

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1. Introduction

1.1. Basic railway definitions

Unlike road vehicles, trains can move from one track to another only at certain places, where special devices (turnouts) are installed.

Let us observe the Fig. 1. The turnout has a mobile part (switch) that sends trains coming from x1 in one of the two possible directions (direct track, also called straight route: x2/diverted track: x3). In the figure the switch is in the diverted track position. In such case there would be no problem if a train would come from x3: it would pass to section x1. But if the train would come from x2 instead, it would ‘trail through a switch set against it’.

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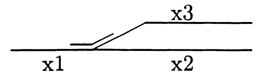

Fig. 1. Layout with one turnout.

If the turnout has a modern spring switch nothing happens: the train passes to section x1 and the spring returns the switch to the original position afterwards. That will be the case considered in this paper. Nothing happens either if the switch is an old-style tramway stub. In other kinds of turnouts the derailment of the train and/or serious damage for the switch can occur.

Note that in this subsection we are only considering one train and we are not thinking about avoiding collisions yet.

Except in uncommon low traffic lines operated by radio (and in yards), traffic is controlled by semaphores and signals. Traditionally the word ‘signal’ is reserved for mechanical devices meanwhile ‘semaphore’ is usually a device with (only) colour-changing lights. Anyway we shall use both words interchangeably in this paper.

1.2. The decision problem

The station master of a station can give clearance simultaneously to more than one train. This action should not allow two trains to collide at any point (in the worse case). Observe that this is not a scheduling problem but a problem of compatibility of permissions.

When there are several trains, signals and turnouts involved, this is not a trivial problem. We shall consider that all trains are allowed to move at the same time in any direction at any speed, unless there is a signal forbidding the movement [4].

1.3. Brief notes about the historical development

Very soon after railway networks began to develop, the first interlocking devices were installed (Saxby, near London, 1859 [11]). Initially they were complicated mechanical equipments, designed to prevent immediately conflicting actions.

In this century electric relays have been used instead. In fact for simple cases they are still being used. But this installations are topology-dependant (i.e., dependant on the layout of the tracks), and very complicated to design.

Since the eighties, high-tech companies, such as Siemens, began to install microcomputer controlled interlocking systems [10,12–15].

1.4. Our approach

Observe that we shall treat here only the ‘logical’ problem (compatibility). This paper includes an improvement of the method explained in [9], where the way the safety of the logical problem is treated is very similar to that developed in [6] to check consistency of KBSs. Here the position of switches, signals and trains is also translated into an ideal of a polynomial residue class ring. In both cases what has to be checked is the degeneracy of an ideal into the whole residue class ring (this is done using Gröbner bases [1]).

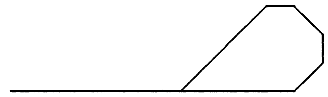

Fig. 2. Reversing loop.

Unlike other more classical approaches like in [2,3,5], there is no translation of the situation into Logic. In this case the steps are:

railway situation → graph → (algebraic) decision model.

2. The associated digraph

Let us remember that trailing through a switch set against is allowed. The digraph will include the information about switches and semaphores.

2.1. Accessibility to the next sections

Four oriented graphs (GD, GS, G^*, G) will be considered. The vertices of the graph are the sections of the line.

Graph GD corresponds to the turnouts and the layout. There is an edge connecting section $\mathrm{x}i$ and section $\mathrm{x}j$ iff one of the following conditions holds:

- Sections $\mathrm{x}i$ and $\mathrm{x}j$ are consecutive in the line (i.e., they are two consecutive sections of a block-system).
- There is a turnout connecting sections $\mathrm{x}i$ and $\mathrm{x}j$ and the switch is in the position that connects sections $\mathrm{x}i$ and $\mathrm{x}j$.
- There is a turnout connecting sections $\mathrm{x}i$ and $\mathrm{x}j$ and the switch is in the position such that it is possible to pass from section $\mathrm{x}i$ to section $\mathrm{x}j$ trailing through this switch set against.

Graph GS corresponds to semaphores. There is an edge connecting section $\mathrm{x}i$ with section $\mathrm{x}j$ iff there is a semaphore controlling the pass from section $\mathrm{x}i$ to section $\mathrm{x}j$ and it forbids such movement.

Therefore, a next section is accessible from another iff the layout and the position of switches makes it possible and the semaphores do not forbid it, i.e., iff there is an edge connecting them in GD but that edge is not in GS. Such graph will be denoted by G^*. It is clear that, if considered as sets of edges, $G^* = GD - GS$.

2.2. General accessibility

But the problem is not as simple, as the possibility of any train moving from the section it occupies to a next one, then to a next one to this second one, to a next one of this third one, etc., has to be taken into account. Think for instance about shunting in a yard. Obviously the solution is to consider the transitive closure of the graph G^* above, to be denoted by G.

Observe that, unlike other approaches, considering directed graphs without considering the direction of the train allows to deal with situations like reversing loops and reversing triangles without problems (Figs. 2 and 3).

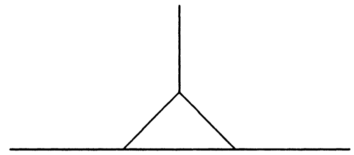

Fig. 3. Reversing triangle.

3. The algebraic model

3.1. Representing the digraph as an ideal

Let us denote the sections by $x_1, x_2, \ldots, x_n$ (polynomial variables). The graph G will be interpreted as a polynomial ideal:

$$I \subseteq \mathbb{Q}[x_1, x_2, \ldots, x_n]$$

that is initialized as $\{0\}$.

That it is possible to move from section x_i to a (next) section x_j – according to the position of the switches and signals – will be represented by including the polynomial:

$$x_i \cdot (x_i - x_j)$$

in the ideal I.

3.2. Preprocessing the ideal

Preprocessing the ideal I is recommended. If it is possible to move both from x_i to x_j and from x_j to x_i then we would have both $x_i \cdot (x_i - x_j)$ and $x_j \cdot (x_j - x_i)$ in I. Polynomial $x_i - x_j$ can substitute both of them for good in this context (see Remark 3).

3.3. Representing the position of trains with a set of polynomials

Trains will be denoted by (different) nonzero integers. If train α is in section x_i then the polynomial $x_i - \alpha$ will be included in the set of polynomials corresponding to the position of the trains, PT. Observe that

- Each section x_i cannot appear more than once, because a section cannot be occupied by more than one train.
- An integer value could appear more than once because a long train could occupy more than one section (for instance $x_l - \alpha$ and $x_m - \alpha$ could be included in PT). Checking the position of the switches under such a train is not considered here (although it was studied with a matrix-based model in [7,8]).

3.4. Checking the situation in the polynomial model

Proposition 1. *Accessibility to the next sections: if train number α is in section x_i, and it is possible to pass from section x_i to a next section x_j, then $x_j - \alpha \in \mathbb{Q}[x_1, x_2, \ldots, x_n]/(I + \langle x_i - \alpha \rangle)$.*

Proof. If train number α is in section x_i, then $x_i - \alpha \in \mathbb{Q}[x_1, x_2, \dots, x_n]/(I + \langle x_i - \alpha \rangle)$, i.e., $x_i = \alpha$ in $\mathbb{Q}[x_1, x_2, \dots, x_n]/(I + \langle x_i - \alpha \rangle)$ and:

1. If it is possible to pass from section x_i to x_j, but it is not possible to pass from section x_j to x_i , then $x_i \cdot (x_i - x_j) \in I$ and therefore $x_j = \alpha$ in $\mathbb{Q}[x_1, x_2, \dots, x_n]/(I + \langle x_i - \alpha \rangle)$.
2. If it is possible to pass both from section x_i to x_j and from x_j to x_i, then $x_i - x_j \in I$ and therefore $x_j = \alpha$ in $\mathbb{Q}[x_1, x_2, \dots, x_n]/(I + \langle x_i - \alpha \rangle)$.

□

So, somehow the value α 'propagates' through the (directed) edges of G^*.

Remark 1. *Let us observe that this really happens not only through the (directed) edges of* G^* *but through those edges in the transitive closure of* G^*, i.e, G *(it can be proven by finite induction).*

Remark 2. *Reciprocally, as the polynomials that generate the ideal I are given by the edges of* G^*, *the value α cannot 'propagate' if there is no (directed) edge linking them in G.*
The following proposition follows in a straightforward way:

Proposition 2. *General accessibility: a train, α, in section x_i, can reach section x_j (according to the position of the switches and signals) iff*:

$$x_j - \alpha \in I + \langle x_i - \alpha \rangle.$$

Using the well known radical membership criterion [1] and Gröbner bases (GB), the previous proposition can be expressed as follows (take into account that the ideals treated here are radical).

Corollary 1. *General accessibility: let t be a new variable, and let us consider the polynomial ring* $\mathbb{Q}[x_1, x_2, \dots, x_n, t]$. *A train, α, in section x_i, can reach section x_j – according to the position of the switches and signals – iff*:

$$\mathrm{GB}(\langle 1 - t \cdot (x_j - \alpha) \rangle + I + \langle x_i - \alpha \rangle) = \{1\}.$$

The previous proposition can also be used to check the safety of a proposed situation. A proposed situation is not safe iff two different trains, α, β (located in sections x_i and x_j, respectively) and a certain section x_k exist such that the two trains can reach section x_k. That is equivalent to $x_k - \alpha, x_k - \beta \in I + \langle \mathrm{PT} \rangle$. As α, β are different numbers, $\alpha - \beta$ is an invertible, and therefore the previous statement is equivalent to the degeneration of the ideal into the whole ring, i.e. to $I + \langle \mathrm{PT} \rangle = \langle 1 \rangle$.

Theorem 1. *Safety: a situation of the switches and signals given by the ideal I and a position of trains given by the ideal* $\langle \mathrm{PT} \rangle$ *is safe iff*:

$$I + \langle \mathrm{PT} \rangle \neq \langle 1 \rangle.$$

Corollary 2. *Safety: a situation of the switches and signaling given by the ideal I and a position of trains given by the ideal* $\langle \mathrm{PT} \rangle$ *is safe iff*:

$$\mathrm{GB}(I + \langle \mathrm{PT} \rangle) \neq \{1\}.$$

Remark 3. *Now the interest of the preprocessing of the ideal I is clear. The* GB *corresponding to* $\{x_i \cdot (x_i - x_j), x_j \cdot (x_j - x_i)\}$ *is something like* $\{x_i \cdot x_j - x_j^2, x_i^2 - x_j^2\}$, *that although leading to the same* *result as $x_i - x_j$ (in this particular application), is far more laborious to handle. This is specially important in case no integer value is 'propagated' through those edges because in such case these polynomials are carried along the subsequent computations.*

4. Maple V.5 implementation

4.1. Data introduction and preprocessing

The code is included in a Maple file that has to be loaded (with a `read` command). It automatically loads Maple V.5's new `Groebner` package.

Then the user has to declare the list of sections. For instance:

```
LV_:=['x.i'$i=1...15];
```

Procedure `inicializa()` initializes the other global variables (the sets of polynomials: `GD_, GS_, PT_`) and has to be executed now. Global variables are used for the sake of brevity.

Procedure `turnout(a,b,c,n)` is used to introduce where the turnouts are and the position of the switches. Trains are sent from section `a` to `b` (through direct track) and to `c` (through diverted track). `0` means direct track and `1` means diverted track. It includes in `GD_` the polynomials corresponding to the new position of the switch and removes those corresponding to the opposite position.

Procedure `adjacent(a,b)` allows to define sections as adjacent (e.g. this is necessary when a line is divided in different sections by a block-system). The polynomial corresponding to the new edge of the graph is included in the set `GD_`.

Procedure `semaphore(a,b,n)` is used to introduce where the semaphores (or signals) are and their colours. Passing from section `a` to `b` is allowed if `n`= 1 (green) and forbidden if `n`= 0 (red). It includes or removes accordingly the corresponding polynomials in `GS_` (let us remember that `GS_` stores the edges forbidden by the semaphores).

Procedure `train(tr,a,n)` introduces the positions of the trains. If `n`= 1 (respectively 0), train number `tr` is declared to be (respectively not to be) in section `a`. It includes the polynomial $a - tr$ in the set `PT_` if `n`= 1, and removes it if `n`= 0.

Finally, `simpliGen(W)` is a tricky Maple procedure that executes the substitution of the preprocessing of Section 3.2 to the set `W`.

4.2. Decision taking procedures

The following boolean procedure applies Corollary 2 in order to check whether the data introduced with the procedures in Section 4.1 are safe or not.

```
isSafe:=proc()
  global GD_, PT_, GS_, LV_;
  evalb (gbasis (simpliGen ((GD_union PT_) minus GS_),
                  tdeg(op (LV_)))
          <> [1])
end;
```

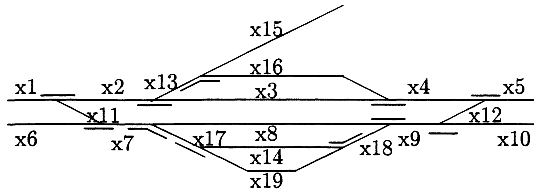

Fig. 4. Track layout of the example.

The following boolean procedure applies Corollary 1 in order to check if train number `tr`, located in section `a`, could reach section `b` (according to the data introduced with the procedures of Section 4.1). Observe that it begins by checking if train `tr` is really in section `a`.

```
isAccessible:=proc(tr,a,b)
  global GD_, PT_, GS_, LV_;
  local t_; #new variable for the radical membership problem
  if not member(a-tr,PT_)
    then print('That train is not there')
    else evalb( gbasis( simpliGen( ({1-t_*(b-tr)} union GD_
                                        union {a-tr}) minus GS_),
                              tdeg(op(LV_), t_))
                  =[1])
    fi;
end;
```

5. Example

The situation in the Fig. 4 will be studied. Observe that neither the semaphores nor the trains have been represented. Timing in a standard 400 MHz Pentium II is specified between brackets.

```
inicializa();
LV_:=['x.i' $ i=1...19];
turnout(x1,x2,x11,0);
turnout(x7,x6,x11,0);
turnout(x7,x8,x17,1);
turnout(x17,x19,x14,0);
turnout(x2,x3,x13,0);
turnout(x13,x15,x16,1);
turnout(x9,x8,x18,0);
turnout(x18,x19,x14,1);
turnout(x4,x3,x16,0);
turnout(x9,x10,x12,0);
turnout(x5,x4,x12,0);
semaphore(x15,x13,1);
semaphore(x16,x13,1);
semaphore(x3,x2,0);
```

```
semaphore(x16,x4,0);
semaphore(x3,x4,0);
semaphore(x8,x7,0);
semaphore(x14,x17,0);
semaphore(x8,x9,1);
semaphore(x14,x18,0);
semaphore(x19,x17,1);
semaphore(x19,x18,0);
train(10,x1,1);
train(7,x3,1);
train(5,x15,1);
train(12,x9,1);
train(9,x14,1);
isSafe();
   false
   (0.295 seconds)
isAccessible(10,x1,x2);
   true
   (0.135 seconds)
isAccessible(10,x1,x3);
   true
   (0.270 seconds)
semaphore(x2,x3,0);
semaphore(x15,x13,0);
isSafe();
   true
   (0.295 seconds)
isAccessible(10,x1,x3);
   false
   (0.620 seconds)
```

6. Conclusions

We think this paper gives a simple but original and ingenious application of Gröbner bases to a non-trivial engineering decision problem. Moreover, the briefness of the code is remarkable (the whole set of procedures is less than 80 lines long).

The authors are now in contact with a railway signaling company to check the commercial interest in a complete environment that would also include trailing detection and have other extensions such as coordination of advanced signaling for low speed with turnouts, security measures, redundancy of computer equipment, etc.

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